Dual Currency Circulation and Monetary Policy

Alessandro Marchesiani† — University of Rome Telma

Pietro Senesi‡ — University of Naples L'Orientale

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Abstract

This paper studies dual money circulation in a closed economy model *à la* Lagos and Wright [16]. It is shown that when both monies can be carried into each match, if they are accepted then they must growth at the same rate. When agents can carry only one money into each match, if both monies are accepted then: (i) the bad money circulates more widely than the good money; (ii) both monies are symmetrically accepted if and only if they grow at the same rate.

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†Faculty of Economics, University of Rome "Telma"; Via Santa Caterina da Siena, 57 - 00186 Rome, Italy; email: `marchesiani@gmail.com`

‡Department of Social Science, University of Naples "L'Orientale", Largo S. Giovanni Maggiore, 30 - 80134 Naples, Italy; email: `psenesi@unior.it`, Phone: +39 081 6909482, Fax: +39 081 6909442

1 Introduction

In many transitional economies, citizens may adopt a dual-payment system by using the foreign (or "good") currency – e.g. the dollar or the euro – in addition to their own locally issued (or "bad") currency as a mean of exchange. In these countries we usually observe that the good currency is not universally accepted in the sense that the great majority of agents make use of the bad currency in transactions ([3], [4], and [12]). This phenomenon rises the following questions. First, under what conditions do the bad and good currencies co-exist? Second, when is the bad currency more widely accepted than the good currency and why? Third, what is the role of monetary policy in each case? The focus of this paper is to address these questions within a third-generation search-theoretic model of money.

Search-theoretic models of monetary economics have become the dominant framework to study dual currency circulation as they formalize the essential role of money explicitly, rather than assuming it exogenously. The search literature on multiple currencies includes, among others, [1], [2], [6], [7], [12], [14], [18], [20], [22], and [24]. All these papers share the common assumption that money is indivisible, and individual holdings of money are bounded at one unit (first- and second-generation models). Some attempts to study multiple-money holdings in a two-currency framework are [3], [5], and [9]. In [3], agents are allowed to hold two units of indivisible money. [9] relax the restriction of money indivisibility, but they assume that goods are indivisible. [5] also study currencies circulation in a divisible-money divisible-good framework, but their model is not analytically tractable since it yields a non-degenerate distribution of money holdings.

The current paper studies monetary policy and dual currency circulation by extending the Lagos and Wright [16] work (hereafter, LW) to two currencies and imperfect information. This extension allows one to derive analytical results while keeping the distribution of money-holdings degenerate (third-generation models). Imperfect information is necessary for introducing restrictions on which money must be used in each type of exchange.

There are a number of papers that are closely related to our work (i.e. [8], [10], [11], and [19]). They all get the distribution of money holdings degenerate, but they never consider the role of information. Furthermore, [15] and [17] introduce imperfect information, but dual currency circulation is never examined in their papers.

The main results of the paper are the following. When both monies can be carried into each match, if they are accepted then they must growth at the same rate. When agents can carry only one money into each match, if both monies are accepted then: (i) the bad money circulates more widely than the good money; (ii) both monies are symmetrically accepted if and only if they grow at the same rate. It is also shown that imperfect information about the authenticity of monies reduces the allocation and welfare.

The paper is organized as follows. Section 2 describes the basic framework and the agents' decision problem. Stationary equilibria are characterized in Section 3. Section 4 introduces imperfect information. Section 5 derives conditions for endogenous acceptability. The conclusions end the paper.

2 The model

The basic set up is LW. Time is indexed by $t \quad \mathbb{N}$ and each period t is divided into two subperiods where different activities take place. There is a $[0, 1]$ *continuum* of infinitely-lived agents and two types of perfectly divisible commodities – general and special goods. Each agent produces a subset and consumes a different subset of the special goods. Specialization is modeled as follows. In the first subperiod, each agent meets someone who produces a good he wishes to consume with probability $\quad \mathbb{R}\,(0, 1/2]$ and meets someone who likes the good he produces with the same probability . With probability $1-2$ an agent has no opportunity to trade. We refer to consumers as buyers and producers as sellers. The specialization of agents over consumption and production of the special good gives rise to a "double coincidence of wants" problem. In contrast to special goods, general goods can be consumed and produced by all agents.

Special goods can only be produced during the first subperiod, while general commodities can only be produced during the second subperiod. In the first subperiod, agents participate in a decentralized market (first market) where each meeting is bilateral and is a random draw from the set of pairwise meetings. In this market the terms of trade are determined by bargaining. In the second subperiod agents produce general goods and can trade in a centralized market (second market).

Agents get utility $u(q)$ from q consumption in the first market, where $u\ (q) > 0$, $u\ \ (q) < 0$, $u\ (0) =$, and $u\ (\ \ ) = 0$. Furthermore, we assume that the elasticity of utility $e(q) = qu\ (q)/u(q)$ is bounded. Producers incur utility cost $c(q)$ from producing q units of output with $c\ (q) > 0$ and $c\ \ (q)\ \ \ 0$. Let q denote the solution to $u\ (q\ ) = c\ (q\ )$. All agents are anonymous. Consequently, trade credit is ruled out and transactions are subject to a *quid pro quo* restriction so there is a role for a medium of exchange ([13] and [23]).

In the second market all agents consume and produce, getting utility $U(x)$ from x consumption, with $U\ (x) > 0$, $U\ (0) =$, $U\ (\ \ ) = 0$ and $U\ \ (x)\ \ \ 0$. Let x be the solution to $U\ (x\ ) = 1$. All agents can produce consumption goods from labor using a linear technology. This implies that all agents will choose to carry the same amount of money out of market 2, independent of their trading history. Agents discount between market 2 and the next-period market 1, but not between market 1 and market 2. This is not restrictive since as in [21] all that matters is the total discounting between one period and the next.

At the beginning of a period, the expected steady state lifetime utility of the representative agent is

$$(1-\ \ )\mathcal{W} = \ \ [u(q) - c(q)] + U(x) - x \tag{1}$$

where q is the quantity consumed by a buyer and produced by a seller in market 1. The solution to the planner problem in an economy without anonymity yields

$$U\ (x\ ) = 1, \tag{2}$$

$$u\ (q\ ) = c\ (q\ ). \tag{3}$$

These are the quantities chosen by a social planner who could force agents to produce and consume.

There are two types $j\ \ \ \{1, 2\}$ of fiat money. It is assumed that two independent central banks exist that control the supply of each money type

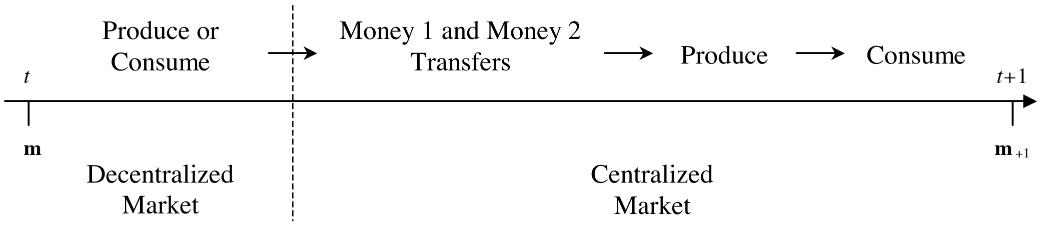

Figure 1: Timing of events

at any time t, $M_{j,t} > 0$. We also assume that $M_{j,t} = \ {}_j M_{j,t-1}$, where $\ {}_j > 0$ is constant and new money of type j is injected, or withdrawn if $\ {}_j < 1$, as lump-sum transfers $\ {}_j M_{j,t-1} = (\ {}_j - 1)\, M_{j,t-1}$ to all agents. We restrict attention to policies where $\ {}_j$, with $\ \mathbb{R}\,(0,1)$ denoting the discount factor. The time subscript t is omitted and shorten $t+1$ to $+1$, etc. in what follows. For notational convenience, if money 1 grows more quickly than money 2 (i.e. $\ {}_1 > \ {}_2$) we refer to money 1 as the "bad" money and money 2 as "good" money, and vice versa.

The timing of events is shown in Figure 1. At the beginning of market 1, bilateral trade of goods begins. In market 2 agents receive lump sum transfers, produce, consume and rebalance their money holdings. The structure of this economy is shown in Figure 2.

At time t, let $\ {}_j = 1/P_j$ be the real price of money j and P_j the price of goods in term of money j in market 2. We study steady state equilibria, where aggregate real money balances are constant over time. We refer to this as stationary equilibrium

$$\ {}_j M_j = \ {}_{j,-1} M_{j,-1} \tag{4}$$

$j \quad \{1,2\}$, which implies that $\ {}_{j,-1}/\ {}_j = M_j/M_{j,-1} = \ {}_j$; the Fisher equation holds, hence it is equivalent to set the nominal interest or inflation.

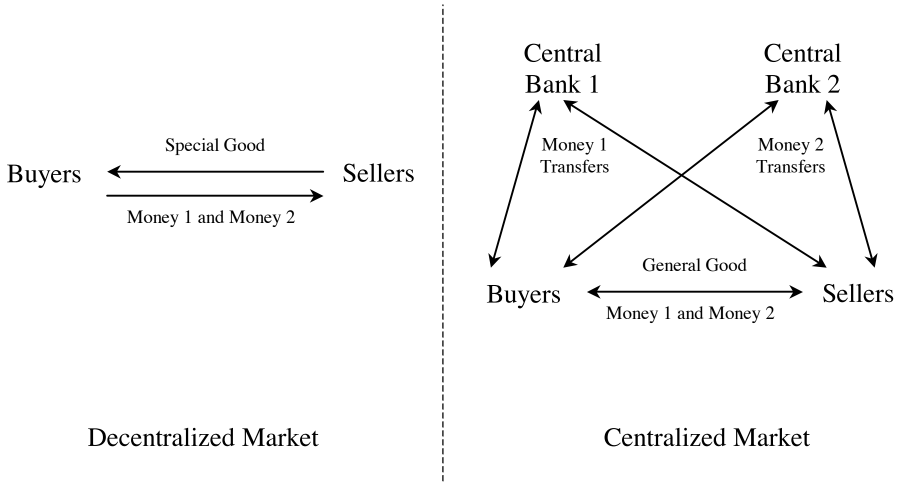

Figure 2: Environment

3 Stationary equilibria

Consider a stationary equilibrium. Let $V(\mathbf{m})$ denote the expected value from trading in market 1 with a portfolio $\mathbf{m}$ $\quad (m_1, m_2)$ of monies. Let $W(\mathbf{m})$ denote the expected value from entering the second market with a portfolio $\mathbf{m}$ of monies. In what follows, we look at a representative period t and work backwards from the second to the first market.

In the second market agents produce h units of good using h hours of labor, receive lump sum transfers, consume x, and adjust their money balances. The real wage per hour is normalized to one. Hence, the representative agent's problem is

$$W(\mathbf{m}) = \max_{x,h,\mathbf{m}_{+1}} \left[U(x) - h + \quad V_{+1}(\mathbf{m}_{+1})\right] \tag{5}$$

such that

$$x + \quad \mathbf{m}_{+1} = h + \quad \mathbf{m} \quad + \quad \mathbf{T} \tag{6}$$

where the vector $\mathbf{m}_{+1}$ $\quad (m_{1,+1}, m_{2,+1})$ is the portfolio of monies taken into period $t+1$ (the term indicates the transpose), $\quad (\;_1, \;_2)$ the real price of monies, and $\mathbf{T}$ $\quad (\;_1M_{1,-1}, \;_2M_{2,-1})$ the lump sum transfers. Eliminate h

from (5) using (6) and get

$$W(\mathbf{m}) = \quad [\mathbf{m} + \mathbf{T}] \\ + \max_{x,\mathbf{m}_{+1}} \; U(x) - x + \quad \mathbf{m}_{+1} + \quad V_{+1}(\mathbf{m}_{+1}) \; . \tag{7}$$

The first order conditions (FOCs) with respect to x and $\mathbf{m}_{+1}$ are

$$U \; (x \;) = 1, \\ V_{j,+1}(\mathbf{m}_{+1}) = \quad _j \tag{8}$$

$j \quad \{1, 2\}$, where the term $V_{j,+1}(\mathbf{m}_{+1})$ is the marginal benefit of taking money j out of market 2 and $\;_j$ is its marginal cost.

There are two main results from (8). First, the quantity of goods x consumed by every agent is equal to the efficient level x such that $U \; (x \;) = 1$. Second, $\mathbf{m}_{+1}$ is independent of $\mathbf{m}$. As a result, the portfolio of monies is degenerate at the beginning of the following period. This is due to the quasi-linearity assumption in (5), which eliminates the wealth effects on money demand in market 2. Agents who bring too much cash into the second market spend some buying goods, while those with too little cash sell goods.

The envelope conditions are

$$W_j(\mathbf{m}) = \quad _j \tag{9}$$

$j \quad \{1, 2\}$, where $W_j(\mathbf{m})$ is the derivative of $W(\mathbf{m})$ with respect to m_j.

An agent who has a portfolio $\mathbf{m}$ of monies at the opening of market 1 has expected lifetime utility

$$V(\mathbf{m}) = \quad [u(q) + W(\mathbf{m} - \mathbf{z}_b)] \\ + \quad [-c(q) + W(\mathbf{m} + \mathbf{z}_s)] \\ + (1 - 2 \;\;) W(\mathbf{m})$$

where $\mathbf{z}_b \quad (z_{1,b}, z_{2,b})$ is the portfolio of monies given up when a buyer and $\mathbf{z}_s \quad (z_{1,s}, z_{2,s})$ is the portfolio of monies received as a seller. From linearity of $W(\mathbf{m})$, expression (7) can be rewritten as

$$W(\mathbf{m}) \quad W(0) + \quad \mathbf{m}$$

which can be used to rewrite the indirect utility function as follows

$$V(\mathbf{m}) = W(\mathbf{m}) + \quad u(q) - \quad \mathbf{z}_b \quad + \quad -c(q) + \quad \mathbf{z}_s \quad . \tag{10}$$

Again, due to linearity of $W(\mathbf{m})$, the Nash bargaining problem in market 1 reduces to

$$\max_{q,\mathbf{z}} \; u(q) - \quad \mathbf{z} \qquad -c(q) + \quad \mathbf{z} \quad ^{1-}$$

such that

$$\mathbf{z} \quad \mathbf{m} \tag{11}$$

where $\quad \mathbb{R}(0,1]$ is the buyer's bargaining power, and $\mathbf{z}_b = \mathbf{z}_s = \mathbf{z}$ the portfolio of monies exchanged in a bilateral meeting. The constraint (11) means that buyers cannot spend more money $\mathbf{z}$ than what they bring into the first market $\mathbf{m}$. The solution to the bargaining problem is

$$\mathbf{z} \quad = g(q) \qquad \frac{u(q)c(q)+(1- \quad)u(q)c(q)}{u(q)+(1- \quad)c(q)} \tag{12}$$

where $g(q) > 0$. Assuming that buyers have all the bargaining power, i.e. $= 1$, equation (12) reduces to $\mathbf{z} \quad = g(q) = c(q)$. If $\mathbf{z} \qquad c(q \ )$ then the buyer exchange $\mathbf{z} < \mathbf{m}$ of his portfolio for the first best quantity q . Otherwise, he gives the seller all of his portfolio, $\mathbf{z} = \mathbf{m}$, in exchange for the quantity q that satisfies $c(q) = \quad \mathbf{z}$. Note that the outcome is independent of the seller's portfolio of monies.

Thus, assuming $= 1$, it holds that

$$q \quad \mathbf{z} \quad = \quad \begin{matrix} q & \text{if} \quad \mathbf{z} \qquad c(q \ ) \\ c^{-1} \qquad \mathbf{z} & \text{if} \quad \mathbf{z} \quad < c(q \ ) . \end{matrix} \tag{13}$$

Using the bargaining solution, the value function (10) can be rewritten as

$$V(\mathbf{m}) = W(\mathbf{m}) + \quad [u(q(\mathbf{m})) - c(q(\mathbf{m}))] . \tag{14}$$

Now, take the differential of (14) with respect to m_j for $j \quad \{1,2\}$ and get

$$V_j(\mathbf{m}) = W_j(\mathbf{m}) + \quad _j \quad \frac{u(q)}{c(q)} - 1 \tag{15}$$

where $q/ \ m_j = \ _j/c \ (q)$ has been used. (Notice that if (11) is binding, then $\mathbf{m} \ = c(q)$.)

By (15), (9), and the second condition in (8) lagged one period, the following holds

$$\frac{_{j,-1}}{} = \ _j \ 1 + \ \frac{u \ (q)}{c \ (q)} - 1 \tag{16}$$

$j \ \ \{1, 2\}$. Directly from the Fisher equation, $1 + i_j = (1 + r)(1 + \ _j)$ with $r = 1/ \ - 1$, we write the equilibrium condition

$$\frac{_{j} -}{} = i_j. \tag{17}$$

The other equilibrium condition is

$$\frac{_{j} -}{} = \ \frac{u \ (q)}{c \ (q)} - 1 \tag{18}$$

where (16) has been used.

Definition 1 *A steady state monetary equilibrium with monies 1 and 2 being accepted in each meeting is a couple* (q, i_j) *satisfying* (17)-(18).

At this point of the analysis the first main result can be introduced:

Proposition 2 *When both monies can be carried into each match, if they are accepted then they must growth at the same rate.*

Proof. Assume both money 1 and money 2 are accepted. Then, (18) must hold, which implies

$$\frac{_{1} -}{} = \frac{_{2} -}{}$$

or, equivalently, $_1 = \ _2$. ■

Following Proposition 2, when two monies can be carried into each match, they differ only in their growth rate. Consequently, both money 1 and money 2 circulate simultaneously if and only if they grow at the same rate.

4 Imperfect information

So far we have assumed that both monies 1 and 2 can be accepted in decentralized meetings. In this section, we relax this hypothesis by assuming that an agent can find himself in two types $j \quad \{1,2\}$ of meetings in the first market: with probability $\ _1$ he is in a meeting where only money 1 can be used for payment (type 1 meeting), while with probability $\ _2 = 1 - \ _1$ he can only use money 2 (type 2 meeting). This is formalized by introducing the assumption that agents are uninformed about the authenticity of money. Assume that each money type can be costlessly counterfeit and that some sellers – or each seller sometimes – cannot detect money j from counterfeits. Under this assumption they will never accept money j in bilateral transactions. For now $\ _1 \quad \mathbb{R}(0,1)$, and is exogenous; we endogenize it below.

The value function of an agent who enters the centralized market with a portfolio $\mathbf{m}$ satisfies (7). The expected lifetime utility for an agent entering market 1 with a portfolio $\mathbf{m}$ of monies is

$$\begin{aligned} V(\mathbf{m}) = & \ _1 \{u(q_1) + W(\mathbf{m} - \mathbf{z}_{1,b}) - c(q_1) + W(\mathbf{m} + \mathbf{z}_{1,s})\} \\ & + \ _2 \{u(q_2) + W(\mathbf{m} - \mathbf{z}_{2,b}) - c(q_2) + W(\mathbf{m} + \mathbf{z}_{2,s})\} \\ & + (1 - 2 \ ) W(\mathbf{m}) \end{aligned} \quad (19)$$

where $\mathbf{z}_{1,b} \quad (z_{1,b}, 0)$, $\mathbf{z}_{2,b} \quad (0, z_{2,b})$, $\mathbf{z}_{1,s} \quad (z_{1,s}, 0)$, and $\mathbf{z}_{2,s} \quad (0, z_{2,s})$. The quantities q_1 and q_2 denote the units of good exchanged in a meeting of type 1 and type 2, respectively. Again, these quantities will be determined by Nash bargaining:

$$\max_{q_j, z_j} [u(q_j) - \ _j z_j] \ [-c(q_j) + \ _j z_j]^{1-} \quad (20)$$

such that

$$z_j \quad m_j \quad (21)$$

$j \quad \{1,2\}$, where $z_j = z_{j,b} = z_{j,s}$. Then, the solution to (20)-(21) is

$$\ _j z_j = g(q_j) \quad \frac{u\ (q_j)c(q_j) + (1- \ )u(q_j)c\ (q_j)}{u\ (q_j) + (1- \ )c\ (q_j)}. \quad (22)$$

Assuming that buyers have all the bargaining power, (22) reduces to $\ _j z_j = g(q_j) = c(q_j)$. Again, if $\ _j z_j \quad c \ q_j$ then the buyer exchange $z_j < m_j$ of money j for the first best quantity q_j. Otherwise, he gives the seller all of his money j (i.e. $z_j = m_j$) in exchange for the quantity q_j that satisfies $c(q_j) = \ _j z_j$. Thus, it holds that

$$q_j(\ _j z_j) = \begin{matrix} q_j & \text{if} \ \ _j z_j \quad c \ q_j \\ c^{-1}(\ _j z_j) & \text{if} \ \ _j z_j < c \ q_j \end{matrix} \ . \quad (23)$$

Using the bargaining solution (23) and linearity of $W(\mathbf{m})$, the value function (19) can be rewritten as

$$\begin{aligned} V(\mathbf{m}) = & \quad {}_1 [u(q_1(m_1)) - c(q_1(m_1))] \\ & + \quad {}_2 [u(q_2(m_2)) - c(q_2(m_2))] + W(\mathbf{m}). \end{aligned} \tag{24}$$

Now, take the differential of (24) with respect to m_j for $j \quad \{1,2\}$ and get

$$V_j(\mathbf{m}) = W_j(\mathbf{m}) + \quad {}_j \; {}_j \; \frac{u(q_j)}{c(q_j)} - 1 \tag{25}$$

where $q_j / \; m_j = \; {}_j / c(q_j)$ has been used. Again, from (9) and the second equation in (8) lagged one period one gets

$$\frac{{}_{j,-1}}{} = \; {}_j \; 1 + \quad {}_j \; \frac{u(q_j)}{c(q_j)} - 1 \quad . \tag{26}$$

Then, using the Fisher equation and taking the steady state of (26), equilibrium conditions are

$$\frac{{}_{j} -}{} = i_j \tag{27}$$

and

$$\frac{{}_{j} -}{} = \quad {}_j \; \frac{u(q_j)}{c(q_j)} - 1 \tag{28}$$

$j \quad \{1,2\}$.

Definition 3 *An equilibrium with imperfect information is a couple* (q_j, i_j) *satisfying* (27)-(28), $j \quad \{1,2\}$, *for given* ${}_j$.

Our second main result can now be established:

Proposition 4 *Imperfect information about counterfeits reduces the allocation and welfare.*

Proof. Assume imperfect information. This implies that (28) holds. Since ${}_j \quad \mathbb{R}(0,1)$, the expression within brackets must be larger, for given ${}_j >$, in an economy with imperfect information than without. Comparison of equations (28) and (18) implies $q_j < q$ for any ${}_j >$, with $j \quad \{1,2\}$. ■

5 Endogenous acceptability

In the previous section, the seller's decision to accept money 1 or money 2 was exogenous. We will now endogenize it. In doing so, we retain the assumption that at most one type of money can be used in a bilateral meeting but relax the hypothesis that agents are uninformed. Before deriving conditions for voluntary acceptability, note that the value of money j depends on the number of agents who bring money j into the first market; but the incentive for agents to bring money j depends in turn on the number of sellers who accept money j.

Recall that the probability that an agent is a buyer or seller is , and thus for any $_1 \quad \mathbb{R}[0,1]$, the expected benefit of bringing money 1 into the first market is

$$(\ _1) \qquad \{[g(q_1(\ _1)) - c(q_1(\ _1))] - [g(q_2(\ _1)) - c(q_2(\ _1))]\} \tag{29}$$

where the expression within brackets is the extra utility from being in a type 1 meeting, as opposed to a type 2 meeting. This expression must be discounted and weighted by the probability of being a seller in market 1.

From the bargaining solution we know that $g(q_j) = c(q_j)$, for $= 1$; note that if the seller's bargaining power is zero (i.e. $= 1$) then his net benefit is zero in each type j of meeting and for any quantity q_j of goods exchanged. This implies $(\ _1) = 0$ always holds. Then, use $_2 = 1 - \ _1$ to solve (28) for $_1$, and get the fraction of agents accepting money 1 in the first market:

$$_1 = \frac{(\ _1 - \ )\ \frac{u(q_2)}{c(q_2)} - 1}{(\ _2 - \ )\ \frac{u(q_1)}{c(q_1)} - 1 + (\ _1 - \ )\ \frac{u(q_2)}{c(q_2)} - 1}. \tag{30}$$

Now, assume that sellers have a strictly positive bargaining power – i.e. < 1. Then, it holds that $g(q_j) > c(q_j)$, directly from (22). So that: (i) $(\ _1) = 0$ iff $q_1 = q_2$; (ii) $(\ _1) > 0$ iff $q_1 > q_2$; and (iii) $(\ _1) < 0$ iff $q_1 < q_2$. (This is a direct consequence of the fact that < 1 implies $g(q) > c(q)$ for any $q < q$.)

To summarize, if < 1 and information is perfect we have three distinct cases. (i) *Only money 1 is accepted*: $(\ _1) > 0$ implies $_1 = 1$ so all agents accept money 1 in bilateral meetings and nobody brings money 2 into the first market. (ii) *Only money 2 is accepted*: $(\ _1) < 0$ implies $\ _1 = 0$. Then money 2 only is used for payment and nobody brings money 1 out of the second market. (iii) *Both money 1 and money 2 are accepted*: $(\ _1) = 0$ implies that a fraction $\ _1 \ \mathbb{R}\,(0,1)$ of agents accept money 1, while a fraction $1 - \ _1$ accept money 2 in transactions.

To determine the equilibrium fraction $\ _1$ for $(\ _1) = 0$, substitute $q_1 = q_2 = q$ into (30) and get

$$\ _1 = \frac{1-}{1+\ _2-2\ }. \tag{31}$$

Thus, the following result can be written:

Proposition 5 *If $\ < 1$, information is perfect, and both monies are accepted (i.e. $(\ _1) = 0$), then the fraction of agents who accept money 1 is increasing in $\ _1$ and decreasing in $\ _2$. The opposite holds for money 2.*

Proof. Take the differential of (31) with respect to $\ _1$ and $\ _2$, and obtain

$$\frac{\ _1}{\ _1} = \frac{\ _2-}{(\ _1+\ _2-2\ )^2} > 0,$$

$$\frac{\ _1}{\ _2} = -\frac{\ _1-}{(\ _1+\ _2-2\ )^2} < 0.$$

Using the fact that $\ _1 = 1 - \ _2$, the rest of the proof (i.e. $\ _2/\ _2 > 0$ and $\ _2/\ _1 < 0$) is straightforward. ■

The next main result can be established:

Proposition 6 *When agents can carry only one money into each match, if both monies are accepted:*
(i) the bad money circulates more widely than the good money;
(ii) money 1 and money 2 are symmetrically accepted if and only if they grow at the same rate.

Proof. Assume that agents can carry at most one type of money into each match. Also assume $\ < 1$, perfect information, and $(\ _1) = 0$. Then (31) must hold. This implies that: (i) $\ _1 > \ _2$ iff $\ _1 > \ _2$, and $\ _1 < \ _2$ iff $\ _1 < \ _2$; and (ii) $\ _1 = \ _2$ iff $\ _1 = \ _2$. ■

When co-existence of monies holds, a bad currency (i.e. a currency with a larger $\ $) should circulate more widely than a good money (i.e. a money with a smaller $\ $). Agents are willing to take the bad money out of the centralized market only if they have a sufficiently higher chance to spend it later in decentralized trades. Namely, bad-money holders are less protected against the inflation tax so they want to spend their cash as soon as they can. Consequently, the higher cost of inflation associated with the bad currency must be counterbalanced by the lower acceptability value of the good money for both currencies to be accepted in transactions. This will make sellers indifferent between accepting the good money or accepting the bad money: the good money provides a better protection against the inflation tax, but the bad money has an higher acceptability value.

6 Conclusions

To be added

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